

Kantowski-Sachs Universe in the Varying Speed of Light Theory

G.S. Khadekar¹ • Arti Ghogre² • Saibal Ray³

Abstract In this work we consider the Kantowski-Sachs (KS) universe in the framework of varying speed of light theory. We present the general solutions of the gravitational field equations with variable speed of light $c(t)$, gravitational coupling parameter $G(t)$ and the decaying vacuum energy $\Lambda(t)$ for the KS model. In the limiting case for the equation of state (EOS) parameter $\gamma = 2$ (stiff fluid with $p = \rho c^2$) and $\gamma = 1$ (dust with $p = 0$), exact solutions of the field equations are obtained. The numerical solutions are also presented for both the cases. Moreover, it is shown that in the limiting case of large time, the mean anisotropy parameter tends to zero for $\gamma = 2$ and $\gamma = 1$. Thus the time variation of the fundamental constants provides an effective mechanism for the isotropization of the KS universe.

Keywords general relativity, Kantowski-Sachs universe, varying speed of light

1 Introduction

Dirac (1938) proposed the large number hypothesis (LNH), motivated by the occurrence of large numbers in the universe. However, in this connection an

exclusive review on the LNH can be obtained in the work of Ray et al. (2007a) for further reading. Inspired by this theory several scientists (Abdel-Rahman 1988, 1992; Beesham 1993; Ray 2004; Ray et al. 2007b; Ray 2008; Mukhopadhyay et al. 2008a,b; Usmani et al. 2008; Ray et al. 2009; Mukhopadhyay et al. 2009, 2010a,b; Ray et al. 2011; Hossein et al. 2012; Ray et al. 2013; Rahaman et al. 2013; Mukhopadhyay et al. 2016) have been intensively investigated problems on the variable gravitational constant and cosmological constant with variable cosmological term Λ in the cosmological as well as astrophysical realm. On the other hand, works have been done on the cosmological models with variable cosmological term within a framework of dissipative thermodynamics as well as in the case of perfect fluid (Vishwakarma 2005; Belinchon 2006, 2008). As G couples geometry to matter, it is reasonable to consider $G = G(t)$ in an evolving universe when one considers $\Lambda = \Lambda(t)$. Many extensions of general relativity with $G = G(t)$ has been made ever since Dirac first consider the possibility of variable G , though none of these theories has gained wide acceptance (Brans & Dicke 1961; Wesson 1978, 1980; Canuto et al. 1977; Weinberg 1971; Norman 1986; Levit 1980; Abdel-Rahman 1990).

The varying speed of light (VSL) cosmologies was proposed by Moffat (1993) as an alternatives to cosmological inflation to provide different basis for resolving the problems of the standard models. He conjectured that a spontaneous breaking of the local Lorentz invariance and diffeomorphism invariance associated with a first order phase-transition can lead to the variation of the speed of light in the early universe. This idea was later on considerably revived by Albrecht & Magueijo (1999) and Barrow (1999, 2000). Barrow showed that the conception of VSL can lead to the solution of flatness, horizon and monopole problems if the speed of light falls at an appropriate rate. Alexander (2000) widely studied the dynamics of VSL in theoretical as

G.S. Khadekar

¹Department of Mathematics, R.T.M. Nagpur University, Mahatma Jyotiba Phule Educational Campus, Amravati Road, Nagpur 440033, Maharashtra, India
gkhadekar@yahoo.com

Arti Ghogre

²Department of Mathematics, Shri Datta Meghe Polytechnic, Wanadogri, Hingna Road, Nagpur 441110, India
arti_ghogre@rediffmail.com

Saibal Ray

³Department of Physics, Government College of Engineering and Ceramic Technology, Kolkata 700010, West Bengal, India
saibal@associates.iucaa.in

well as empirical context. It has in particular been speculated that VSL offers new paths of solving the problems of the standard Big Bang cosmology which are distinct from their resolutions in context of the inflationary paradigm (Guth 1981) or the pre-Big Bang scenario of low energy string theory (Veneziano 1997). Moreover, in contrast to the case of inflationary Universe, VSL may provide an explanation for the relativistic smallness of the various physical constants today.

In connection to VSL theory Barrow (1999) introduced a Machian scenario in which $c = c_0 a^n$, where a is the scale factor. This has significant advantages to the phase-transition scenario in which the speed of light changes suddenly from c^- to c . On the other hand, Albrecht & Magueijo (1999) have investigated possible consequences of a time variation in the velocity of light in vacuum. The Einstein field equations for FRW spacetime in the VSL theory have been solved by Barrow (1999, 2000) for anisotropic model, who also obtained the rate of variation of the speed of light required to solve flatness and cosmological constant problems. Some other mentionable works in this field are as follows: (i) By assuming energy conservation of observed matter Gopakumar & Vijayagovindam (2001) have solved the flatness cosmological constant problem with varying speed of light c , gravitational coupling strength G and cosmological parameter Λ , (ii) Chimento & Jakubi (2001) have found exact constant solutions for cosmological density parameter using generalization of general relativity that incorporates a cosmic time variation of velocity of light in vacuum and Newtonian gravitation constant, and (iii) Belinchon (2005, 2006, 2007) studied perfect fluid Bianchi type I model with variable G , c and Λ .

Besides all the above mentioned major works on VSL theory it is observed that some authors (Harko & Mak 1999; Shojaie & Farhoudi 2006) have proposed a new generalization of general relativity which also allows arbitrary changes in the speed of light c and the gravitational constant G . However, this has been done in such a way that variation in the speed of light introduces corrections to the curvature tensor in the Einstein equations in the cosmological frame. Harko & Mak (1999) considered the evolution and dynamics of Bianchi type I and V Universe and obtained exact solutions of the gravitational field equations in a small time scale limit.

The purpose of the present paper is to extend the results previously obtained in the framework of the homogeneous and isotropic FRW cosmological models to the case of anisotropic KS universe. In essence we carried out here an investigation to highlight specific features of KS Universe which basically represents dust solutions to the Einstein field equation and are a widely used family of inhomogeneous cosmological models. Basically,

we have generalized the work of Harko & Mak (1999) by considering that the constants are functions of the volume scale factor and obtained the solutions of the field equations under the framework of VSL theory in the small time limit. The paper is organized as follows: The field equations for KS model are written down in Sec. 2. In Sec. 3 we have obtained exact solutions of the field equations for VSL model corresponding to specific time variation law of constants. In Sec. 4, we conclude our results.

2 The Einstein field equations

We consider the KS Universe in the framework of Einstein's general relativity. As an additional condition we impose on the physical constants some restrictions as advocated by the LNH and VSL theory. The line element for KS Universe is given by

$$ds^2 = -c^2(t)dt^2 + a_1^2(t)dr^2 + a_2^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where a_1, a_2 are the scale factors.

The Einstein field equations take the usual form

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi G(t)}{c^4(t)}T_{ij} + \Lambda(t)g_{ij}. \quad (2)$$

The energy momentum tensor can be written as

$$T_{ij} = (p + \rho c^2)u_i u_j + p g_{ij}, \quad (3)$$

where u^i ($i = 0, 1, 2, 3$) be the four velocity, p and ρ are respectively the fluid pressure and energy density.

For the KS metric (1), the Einstein field Eq. (2) can be written as

$$\frac{\dot{a}_2^2}{a_2^2} + \frac{2\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{c^2}{a_2^2} = 8\pi G\rho + \Lambda c^2, \quad (4)$$

$$\frac{2\ddot{a}_2}{a_2} + \frac{\dot{a}_2^2}{a_2^2} - \frac{2\dot{a}_2\dot{c}}{a_2c} + \frac{c^2}{a_2^2} = -\frac{8\pi Gp}{c^2} + \Lambda c^2, \quad (5)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{\dot{a}_2\dot{c}}{a_2c} - \frac{\dot{a}_1\dot{c}}{a_1c} = -\frac{8\pi Gp}{c^2} + \Lambda c^2. \quad (6)$$

We assume that the thermodynamic pressure p of the cosmological fluid obeys a linear equation of state

$$p = (\gamma - 1)\rho c^2(t), \quad (7)$$

where equation of state parameter $\gamma = \text{constant}$ and $1 \leq \gamma \leq 2$.

The conservation equation gives

$$\dot{\rho} + \left(\frac{\dot{a}_1}{a_1} + \frac{2\dot{a}_2}{a_2} \right) \gamma \rho = -\frac{c^2 \dot{\Lambda}}{8\pi G} - \frac{\dot{G}}{G} \rho + \frac{2\dot{c}}{c} \rho. \quad (8)$$

We assume that $(T_{i,j}^j) = 0$, which leads to the following two equations:

$$\dot{\rho} + \left(\frac{\dot{a}_1}{a_1} + \frac{2\dot{a}_2}{a_2} \right) \gamma \rho = 0, \quad (9)$$

$$-\frac{c^2 \dot{\Lambda}}{8\pi G} - \frac{\dot{G}}{G} \rho + \frac{2\dot{c}}{c} \rho = 0. \quad (10)$$

For later convenience we introduce the following variables:

$$V = a_1 a_2^2, \quad (11)$$

$$H_i = \frac{\dot{a}_i}{a_i}, \quad (12)$$

$$H = \frac{1}{3}(H_1 + 2H_2). \quad (13)$$

We further define

$$\Delta H_i = H_i - H, \quad (14)$$

where $i = 1, 2, 3$ and $H_2 = H_3 = \frac{\dot{a}_2}{a_2}$, the factors V , H_i and H being the volume scale factor, directional Hubble parameters and mean Hubble parameter respectively.

We obtain

$$H = \frac{\dot{V}}{3V}. \quad (15)$$

In addition to this we introduce the physical quantities related to cosmology as follows:

$$\theta = 3H, \quad (16)$$

$$A = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2 = \frac{1}{3} \sum_{i=1}^3 \frac{(H_i - H)^2}{H^2}, \quad (17)$$

$$\sigma^2 = \frac{1}{2} \sigma_{ik} \sigma^{ik} = \frac{1}{2} \left(\sum_{i=1}^3 H_i^2 - 3H^2 \right) = \frac{3AH^2}{2}, \quad (18)$$

$$q = -H^{-2}(\dot{H} + H^2) = \frac{d}{dt} \left(\frac{1}{H} \right) - 1, \quad (19)$$

where θ , A , σ^2 and q are the scalar expansion, mean anisotropy parameter, shear scalar and deceleration parameter respectively.

Note that $A = 0$ for isotropic expansion. Moreover, the signature of the deceleration parameter indicates how the Universe expands. In fact the positive sign corresponds to ‘standard’ deceleration model whereas a negative sign indicates an accelerating Universe.

3 An exact solution for the KS universe

In this section we find the exact solutions of the field equations.

From Eq. (5) and (6), we get

$$\frac{\dot{a}_2^2}{a_2^2} + \frac{\ddot{a}_2}{a_2} + \frac{c^2}{a_2^2} - \frac{\dot{a}_2 \dot{c}}{a_2 c} + \frac{\dot{a}_1 \dot{c}}{a_1 c} = \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\ddot{a}_1}{a_1}. \quad (20)$$

Adding Eq. (4) and (5) and using Eq. (20) in it, we obtain

$$\frac{1}{V} \frac{d}{dt} (V H_1) - \frac{\dot{c}}{c} H_1 = 4\pi G \left(\rho - \frac{p}{c^2} \right) + \Lambda c^2. \quad (21)$$

Also adding Eq. (4) and (6) and using Eq. (20), we get

$$\frac{1}{V} \frac{d}{dt} (V H_2) - \frac{\dot{c}}{c} H_2 = 4\pi G \left(\rho - \frac{p}{c^2} \right) + \Lambda c^2 - \frac{c^2 a_1}{V}. \quad (22)$$

From Eqs. (21) and (22) with the help of Eq. (12) one can get

$$3\dot{H} + H_1^2 + 2H_2^2 - 3\frac{\dot{c}}{c} H = -4\pi G \left(\rho + 3\frac{p}{c^2} \right) + \Lambda c^2. \quad (23)$$

Again adding Eq. (21) and (22), we get

$$\begin{aligned} \frac{1}{V} \frac{d}{dt} (V [H_1 + 2H_2]) - \frac{\dot{c}}{c} (H_1 + 2H_2) \\ = 12\pi G \left(\rho - \frac{p}{c^2} \right) + 3\Lambda c^2 - \frac{2c^2 a_1}{V}. \end{aligned} \quad (24)$$

Now using Eq. (13) and (15) in Eq. (24), with (7) we get

$$\ddot{V} - \dot{V} \frac{\dot{c}}{c} = 3\Lambda V c^2 + 12\pi G (2 - \gamma) \rho V - 2a_1^2 c^2. \quad (25)$$

Now we assume that the ‘constants’ G , c and Λ are decreasing functions of time and to describe their variations we use the following simple phenomenological law given by (Harko & Mak 1999)

$$G = G_0 + \frac{G_1}{V^\alpha}, \quad \Lambda = \Lambda_0 + \frac{\Lambda_1}{V^\beta}, \quad c = c_0 + \frac{c_1}{V^\eta}. \quad (26)$$

where $G_0 > 0, G_1 \geq 0, \Lambda_0 > 0, \Lambda_1 \geq 0, c_0 > 0, c_1 > 0, \alpha > 0, \beta > 0$ and $\eta > 0$ are all constants.

For $t \rightarrow 0$, V is extremely small, then

$$G \approx \frac{G_1}{V^\alpha}, \quad \Lambda \approx \frac{\Lambda_1}{V^\beta}, \quad c \approx \frac{c_1}{V^\eta}. \quad (27)$$

From the conservation Eq. (9), ρ can be expressed as

$$\rho = \rho_0 V^{-\gamma}, \quad (28)$$

where ρ_0 is constant of integration.

After using Eq. (27) and (28) in Eq. (10), we get the consistency conditions relating the constants $\alpha, \beta, \gamma, \eta, G_1, \Lambda_1$ and c_1 as

$$\alpha - \beta + \gamma - 2\eta = 0, \quad (29)$$

$$\alpha - 2\eta = \frac{-c_1^2 \Lambda_1 \beta}{8\pi G_1 \rho_0}. \quad (30)$$

Hence Eq. (29) and (30), yield

$$\beta = \left(1 + \frac{c_1^2 \Lambda_1}{8\pi G_1 \rho_0}\right)^{-1}. \quad (31)$$

Now by using Eq. (27) and (28) in Eq. (25), we get

$$V\ddot{V} + \eta\dot{V}^2 = \frac{3c_1^2 \Lambda_1}{V^{2\eta+\beta-2}} + \frac{12\pi G_1 \rho_0 (2-\gamma)}{V^{\alpha+\gamma-2}} - \frac{2a_1 c_1^2}{V^{2\eta-1}} \quad (32)$$

From Eq. (32), after first integration we get

$$\dot{V}^2 = V^{-2\eta} F(V), \quad (33)$$

where

$$F(V) = \left(\frac{6c_1^2 \Lambda_1}{2-\beta} V^{2-\beta} + \frac{24\pi G_1 \rho_0 (2-\gamma)}{2\eta-\alpha-\gamma+2} V^{(2\eta-\alpha-\gamma+2)} + C \right). \quad (34)$$

where $C = -4c_1^2 \int a_1 dV + c_2$ and $c_2 > 0$ is a constant of integration.

From Eqs. (21) and (22) after integration, we get

$$\log[V(H_1 - H_2)] = \log c + \log K, \quad (35)$$

where $K = \exp \left[\int \frac{c^2 a_1}{V(H_1 - H_2)} dt \right]$.

We note that Obukhov et al. (2006) considered $K = \int a_1 dt$ in the conventional Einstein's theory. In our case the value of K is similar to this value for $\alpha = \beta = \eta = 0$ and $c_1 = 1$.

After simplification Eq. (35) provides

$$H_1 = H + \frac{2}{3} \frac{Kc}{V}, \quad (36)$$

and

$$H_2 = H - \frac{Kc}{3V}. \quad (37)$$

If we substitute Eqs. (36) and (37) in Eq. (23), the with the help of Eqs. (29) and (30) we get

$$\frac{c_1^2 \Lambda_1 \beta [\gamma(\eta + \beta) - 2\eta + 2]}{(\beta - 1)(\beta - 2)V^{(2\eta+\beta-2)}} - \frac{2}{V^{2\eta}} \left(a_1 c_1^2 V + \frac{c_2}{3} \right) + \frac{8c_1^2}{3V^{2\eta}} \int a_1 dV + \frac{2}{3} K^2 = 0. \quad (38)$$

For $\alpha = \beta = \eta = c_2 = 0$ and $c_1 = 1$, the above equation reduces to the equation of the form

$$-2a_1 V + \frac{8}{3} \int a_1 dV + \frac{2}{3} \left(\int a_1 dt \right)^2 = 0,$$

as obtained earlier by Obukhov et al. (2006) in the general theory of relativity.

By taking $V \geq 0$, as a parameter, we can obtain the general solution for KS model with physical quantities as follows:

$$t - t_0 = \int \frac{V^\eta}{F(V)^{1/2}} dV, \quad (39)$$

$$\theta = 3H = \frac{F(V)^{1/2}}{V^{\eta+1}}, \quad (40)$$

$$a_1 = V^{1/3} a_{01} \exp \left[\frac{2}{3} K c_1 \int \frac{dV}{V F(V)^{1/2}} \right], \quad (41)$$

$$a_2 = V^{1/3} a_{02} \exp \left[\frac{-1}{3} K c_1 \int \frac{dV}{V F(V)^{1/2}} \right], \quad (42)$$

$$A = \frac{2K^2 c_1^2}{F(V)}, \quad (43)$$

$$\sigma^2 = \frac{K_1^2 c_1^2}{3V^{2(\eta+1)}}. \quad (44)$$

$$q = 3\eta + 2 + \frac{3V}{2F(V)} \times$$

$$\left[-6\Lambda_1 c_1^2 V^{1-\beta} + 24\pi G_1 \rho_0 (\gamma - 2) V^{(2\eta-\alpha-\gamma+1)} + 4c_1^2 a_1 \right], \quad (45)$$

where t_0 , a_{01} and a_{02} are constants of integration.

3.1 Case I : $\gamma = 2$

In this case for extremely small V , Eq. (34) can be written as

$$F(V) = \frac{6c_1^2\Lambda_1}{2-\beta}V^{2-\beta} + B_1, \quad (46)$$

where B_1 is constant.

Using Eq. (46) in Eq. (33) we get

$$\dot{V}^2 = V^{-2\eta} (\alpha_0 V^{2-\beta} + B_1), \quad (47)$$

where $\alpha_0 = \left(\frac{6c_1^2\Lambda_1}{2-\beta}\right)$, $\beta \neq 2$.

After integrating Eq. (47) with the condition $\alpha_0 V^{2-\beta} \gg B_1$ and $t_0 = 0$, it gives

$$V \approx t^{\frac{2}{2\eta+\beta}} \implies H \approx \frac{1}{t}. \quad (48)$$

Hence the scale factors a_1 and a_2 can be obtained as

$$a_i \approx t^{\frac{2}{3(2\eta+\beta)}}, \quad (49)$$

for $i = 1, 2$.

Using Eqs. (43) and (48) we get

$$A \approx t^{\frac{2(\beta-2)}{2\eta+\beta}}, \quad \beta < 2. \quad (50)$$

$$\sigma^2 \approx \left(\frac{1}{t^{\frac{4(\eta+1)}{2\eta+\beta}}} \right). \quad (51)$$

$$q = 3 \left(\eta + \frac{\beta}{2} \right) - 1. \quad (52)$$

Thus q tends to a constant value. The analogous solutions for extremely small value of V are discussed by Harko & Mak (1999) for Bianchi type I model.

3.2 Case II : $\gamma = 1$

Using Eq. (34) with $\Lambda_1 = 0$ for extremely small value of V , we get

$$F(V) = b_0 V^{2\eta-\alpha+1} + B_3, \quad (53)$$

where $b_0 = \left(\frac{24\pi G_1 \rho_0}{(2\eta-\alpha+1)}\right)$ and $B_3 = \text{constant}$.

Hence using Eq. (53) in Eq. (33), we get

$$\dot{V}^2 = V^{-2\eta} (b_0 V^{2\eta-\alpha+1} + B_3). \quad (54)$$

After integrating Eq. (54) with the condition $b_0 V^{2\eta-\alpha+1} \gg B_3$ and $t_0 = 0$, it gives

$$V \approx t^{\frac{2}{3-\alpha}} \implies H \approx \frac{1}{t}. \quad (55)$$

The other variables can be calculated straightforwardly as follows:

$$a_i \approx t^{\frac{2}{3(3-\alpha)}}, \quad \alpha \neq 3, \quad (56)$$

for $i = 1, 2$.

$$A \approx t^{\frac{-2(2\eta+1-\alpha)}{(3-\alpha)}}, \quad \alpha \neq 3. \quad (57)$$

$$\sigma^2 \approx t^{\frac{-4(\eta+1)}{(3-\alpha)}}, \quad \alpha \neq 3. \quad (58)$$

$$q = \frac{3}{2}(\alpha - 1) + 2. \quad (59)$$

The deceleration parameter is always positive when $\alpha > -1/3$ and negative when $\alpha < -1/3$.

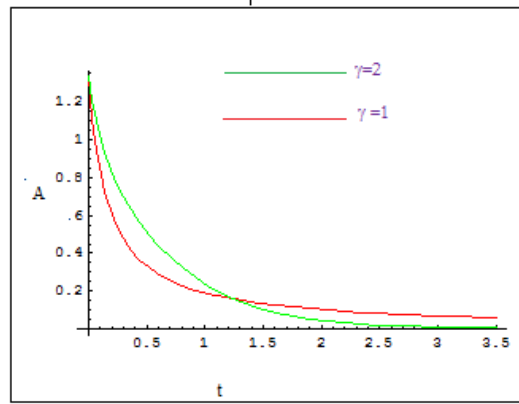


Fig. 1 Variation with respect to time of the mean anisotropy parameter for KS model with perfect cosmological fluid for $\gamma = 2$ (green line) and $\gamma = 1$ (red line), in the small time limit. The constants are chosen as: $\beta = 1, c_1 = 1, C = 1, \Lambda_1 = 1$

4 Conclusion

In this paper we have generalized the work of Harko & Mak (1999) for KS model in the framework of VSL theory. We have obtained the solutions for KS model in a small time limit in which variables G , c and Λ are functions of the volume scale factor. For extremely small value of V and $\gamma = 2$, with the condition $\alpha_0 V^{2-\beta} \gg B_1$, $F(V) \propto V^{2-\beta}$ with $t_0 = 0$, it is observed that $V \propto t^{\frac{2}{2\eta+\beta}}$ and hence, the expansion of the early Universe is of the form of power law of the expansion. The mean anisotropy will increase for $\beta > 2$, and KS spacetime will not end in isotropic state in large time limit. However, for $\beta < 2$, the

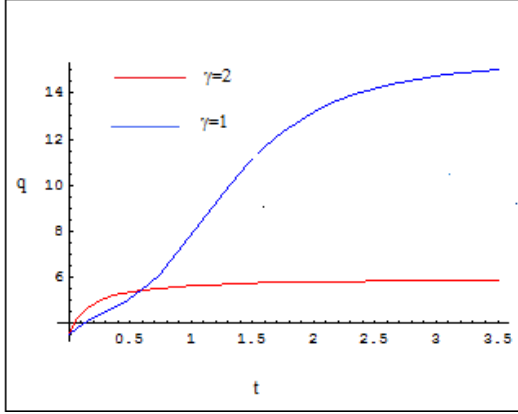


Fig. 2 Variation with respect to the time of the deceleration parameter q for stiff fluid $\gamma = 2$ (Red line), $\gamma = 1$ (blue line) and in the small V limit. The constants are chosen as: $\beta = 1, \eta = 1/2, c_1 = 1, C = 1, \Lambda_1 = 1, 8\pi G_1 \rho_0 = 1, 4c_1^2 a_1 = 1$.

mean anisotropy tends to 0 in the large time limits, thus the KS type VSL cosmology is providing an effective mechanism for the isotropization of the Universe. The evolution of the anisotropic flat Universe is generally non-inflationary, with the deceleration parameter $q > 0$, and tending, for large times, to a constant value given in the Eq. (52). The deceleration parameter is given by $q = 3(\eta + \beta/2) - 1$, which is always positive when $\beta > (2/3 - 2\eta)$ and negative when $\beta < (2/3 - 2\eta)$.

Similarly, for the case $\gamma = 1$ with $\Lambda_1 = 0$ we have shown that $V \propto t^{\frac{2}{3-\alpha}}$ as $t \rightarrow 0$. Hence again the expansion of the early Universe is of the form of power law expansion. For $\alpha > 3$, the mean anisotropy will increase for large times limit and KS space time will not end in an isotropic state, whereas for $\alpha < 3$, the mean anisotropy tends to 0, thus the KS type VSL cosmology becomes isotropic. Moreover, the deceleration parameter is given by $q = \frac{3}{2}(\alpha - 1) + 2$, which is always positive for $\alpha > 1$.

The time variation of mean anisotropic parameter A and deceleration parameter q for KS spacetime are plotted with respect to the time as shown in the Figs. 1 and 2.

It have argued by Damour et al. (1988) that the existence of cosmological constant Λ or variable G is not in conflict with observational determination of the age of the Universe or with some astrophysical data. The dynamics and evolution of the Universe is essentially determined by the values of the constants α , β , and η describing the time variations of G , c and Λ and which are arbitrary in this model.

However, we would like to conclude here with a final comment that Ghosh et al. (2012) investigated the possible variation of c in the context of the present ac-

celerating Universe as discovered through SN Ia observations and show that variability of c is not permitted under the variable Λ models. This obviously impose an observational constraint on the theoretical speculation of VSL theory, however at the same time allows variation of Λ in terms of *dark energy* via the background platform of LNH as put forward by Dirac (1938).

Acknowledgement

The authors are thankful to Prof. Sriram for useful discussions. This work is partially supported by University Grant Commission, New Delhi under the Special Assistance Programme No. F. 510/1/DRS/2011(SAP-I). The authors (GSK and AG) thank for the Inter University Center for Astronomy and Astrophysics (IUCAA), Pune for providing the necessary literature during visit under the SAP programme. Also SR is thankful to the authority of Inter-University Center for Astronomy and Astrophysics, Pune, India as well as The Institute of Mathematical Sciences, Chennai, India for providing Associateship under which a part of this work was carried out.

References

- Abdel-Rahman, A.M.M.M.: Nuo. Cem. B **102**, 225 (1988)
- Abdel Rahman, A.M.M.M.: Gen. Rel. Grav. **22**, 655 (1990)
- Abdel-Rahman, A.M.M.M.: Phys. Rev. D **45**, 3497 (1992)
- Albrecht, A., Magueijo, J.: Phys. Rev. D **59**, 043516 (1999)
- Alexander, S.H.S.: J. High Energy Phys. **11**, 017 (2000)
- Barrow, J.D.: Phys. Rev. D **59**, 043515 (1999)
- Barrow, J.D.: Gen. Rel. Grav. **32**, 1111 (2000)
- Beesham, A.: Gen. Rel. Grav. **26**, 159 (1993)
- Belinchon, J.A.: Astrophys. Space Sci. **299**, 341 (2005)
- Belinchon, J.A.: Astro Phys. Space Sci. **302**, 161 (2006)
- Belinchon, J.A.: Astrophys. Space Sci. **315**, 311 (2007)
- Belinchon, J.A.: Int. J. Mod. Phys. **12**, 25 (2008)
- Brans, C. and Dicke, R. H.: Phys. Rev. **925**, 297 (1961)
- Canuto, V., Adams, R. H., Hsieh, S. H. and Tsiang, E.: Phys. Rev. **D16**, 1643 (1977)
- Chimento, L.P., Jakubi, A.S.: Int. J. Mod. Phys. D **10**, 107 (2001)
- Damour, T., Gibbons, G.W., Taylor, J.H.: Phys. Rev. Lett. **61**, 115 (1988)
- Dirac, P.A.M.: Proc. R. Soc. Lond. A **165**, 199 (1938)
- Ghosh, P.P., Mukhopadhyay, U., Ray, S.: Astrophys. Space Sci. **337**, 509 (2012)
- Gopakumar, P., Vijayagovindam, G.V.: Mod. Phys. Lett. A **16**, 957 (2001)
- Guth, A.H.: Phys. Rev. D **23**, 347 (1981)
- Harko, T., Mak, M.M.: Int. J. Mod. Phys. D **8**, 607 (1999)
- Hossein, S.M., Rahaman, F., Naskar, J., Kalam, M., Ray, S.: Int. J. Theor. Phys. **21**, 1250088 (2012)
- Levit, L.S.: Lett. Nuovo Cimento **29**, 23 (1980)
- Moffat, J.W.: Found. Phys. **23**, 411 (1993)
- Mukhopadhyay, U., Ray, S., Dutta Choudhury, S.B.: Int. J. Mod. Phys. D **17**, 301 (2008a)
- Mukhopadhyay, U., Ray, S., Dutta Choudhury, S.B.: Mod. Phys. Lett. A **23**, 3187 (2008b)
- Mukhopadhyay, U., Ray, S.: N.B.U. Math. J. **II** 51 (2009)
- Mukhopadhyay, U., Ray, P.C., Ray, S., Dutta Choudhury, S.B.: Int. J. Mod. Phys. D **18**, 389 (2009)
- Mukhopadhyay, U., Ray, S., Rahaman, F.: Int. J. Mod. Phys. D **19**, 475 (2010a)
- Mukhopadhyay, U., Ghosh, P.P., Ray, S.: Int. J. Theor. Phys. **49** 1622 (2010b)
- Mukhopadhyay, U., Ray, S., Chakraborty, I., Usmani, A.A.: Int. J. Theor. Phys. **55**, 388 (2016)
- Norman, E. B.: Am. J. Phys. **54**, 317 (1986)
- Obukhov, V.V., Makarenko, A.N., Ostrin, K.E.: J. Phys. A: Math. Gen. **39**, 6635 (2006)
- Rahaman, F., Ray, S., Islam, S.: Astrophys. Space Sci. **346**, 245 (2013)
- Ray, S., Mukhopadhyay, U., Meng, X.-H.: Gravit. Cosmo. **13**, 142 (2007a)
- Ray, S., Bhadra, S.: Phys. Lett. A **322**, 150 (2004)
- Ray, S., Mukhopadhyay, U., Dutta Choudhury, S.B.: Int. J. Mod. Phys. D **16**, 1791 (2007b)
- Ray, S., Bhadra, S., Mohanti, G.: Astrophys. Space Sci. **315**, 341 (2008)
- Ray, S., Rahaman, F., Mukhopadhyay, U.: Int. J. Mod. Phys. D, **18** 781 (2009)
- Ray, S., Khlopov, M., Ghosh, P.P., Mukhopadhyay, U.: Int. J. Theor. Phys. **50** 939 (2011)
- Ray, S., Mukhopadhyay, U., Rahaman, F., Sarkar, R.: Int. J. Theor. Phys. **52**, 4524 (2013)
- Shojaie, H., Farhoudi, M.: Can. J. Phys., **84**, 933 (2006)
- Usmani, A.A., Ghosh, P.P., Mukhopadhyay, U., Ray, P.C., Ray, S.: Mon. Not. Roy. Astron. Soc. Lett. **386** L92-L95 (2008)
- Veneziano, G.: Phys. Lett. B **406**, 297 (1997)
- Vishwakarma, R.G.: Gen. Relativ. Gravit. **37**, 1305 (2005)
- Weinberg, S.: Gravitation and Cosmology, Wiley, NewYork (1971)
- Wesson, P.S.: Cosmology and Geophysics, (Oxford Univ. Press, 1978)
- Wesson, P.S.: Gravity, Particle and Astrophysics (D. Reidel, Dordrecht, 1980)